Parallel-Source Correspondence in the Hypersonic Slender-Body Theory

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Theme

THE theme is to show that general one-to-one correspondence relations between parallel and source flows can be derived in the hypersonic slender-body theory, for the case that the ratio of the specific heats γ of a gas is equal to 2. It is shown that the equations of motion and the boundary conditions are the same for both flows in the transformed, unified coordinates and flow variables. In particular, it is shown that if the pressure coefficient C_p along the body surface is not too sensitive to the difference of γ, then C_p in the parallel and in the source flows with different values of γ can be correlated with each other by a simple conversion relation. Theoretical pressure distributions obtained by applying the present correspondence analogy for several bodies are compared with experimental results, showing good comparisons for power-law bodies, and some differences for a hemisphere cylinder.

Contents

In a recent paper,[1] the authors showed that the dominating equations in the transformed similarity coordinates for the hypersonic source flow over slender power-law nosed cones are essentially the same as the equations for parallel flow over power-law bodies, provided that the ratio of specific heats is equal to 2. The present paper deals with the problem in a more general form: the body shapes are no longer restricted to power-law nosed cones, and the essential variables and parameters which yield one-to-one correspondence between source and parallel flows are precisely determined. Although this correspondence is exactly true only if $\gamma=2$, the present analogy should also be useful in actual pressure problems if the pressure coefficient is insensitive to the difference of γ^2.

The relations between parallel flow and corresponding source flow are shown in the generalized, tabulated form in Table 1 and in Fig. 1.

The relations in the table show that, when $\gamma=2$, the hypersonic flowfield for slender bodies having the same functional expression $Y_b=f(X)$ are governed completely by the same equations and boundary conditions in the transformed variables. Thus, under the condition that $\gamma=2$ and that the viscous effects are ignored, if any analytical or numerical result for a body shape with the functional form of $y_b/L=f(x/L)$ in a parallel flow is obtained at a given freestream Mach number M_N, the corresponding result for a modified body shape $\theta_b=f(1-s^{-1})$ in a source flow with the freestream nose Mach number of M_N can be easily obtained by a simple conversion, and vice versa.

Further, although the value $\gamma=2$ in the foregoing correspondence relations is different from that of actual gases, if the pressure coefficient along the body surface is insensitive to such a difference of γ, then the value of $c_p(x/L; M_N, \gamma)$ for the parallel flow with γ, could be related to the pressure coefficient $c_{pl}(1-s^{-1}; M_N, \gamma_l)$ of the

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Table 1 Generalized relations between parallel and source flows

A. Parallel flow	B. Source flow, $\gamma=2$
Coordinates	
Cylindrical (x,y)	Spherical (r,θ)
Equations of motion	
Same as Ref. 2	Same as Ref. 1
Freestream, as $M_N \gg 1$	
$\left.\begin{array}{l} u_l=u_N,\ p_l=p_N \\ \rho_l=\rho_N,\ M_l=M_N \end{array}\right\}$ (1A)	$\left.\begin{array}{l} u_l=u_N,\ p_l=p_N s^{-4} \\ \rho_l=\rho_N s^{-2},\ M_l=M_N s \end{array}\right\}$ (1B) where $s\equiv r/r_N$
$\left.\begin{array}{l} \text{body: } y=y_b(x) \\ \text{shock: } y=y_w(x) \\ \text{shock angle: } \sigma=\dfrac{dy_w}{dx}\ll 1 \end{array}\right\}$ (2A)	$\left.\begin{array}{l} \theta=\theta_b(s) \\ \theta=\theta_w(s) \\ \sigma=s\dfrac{d\theta_w}{ds}\ll 1 \end{array}\right\}$ (2B)
Unified coordinates (X, Y)	
$\left.\begin{array}{l} X\equiv x/L,\quad Y\equiv y/L \\ \text{shock } Y=Y_w(X) \end{array}\right\}$ (3A)	$\left.\begin{array}{l} X\equiv 1-s^{-1},\quad Y\equiv\theta \\ Y=Y_w(X) \end{array}\right\}$ (3B)
Hypersonic shock parameter	
$\bar{\chi}\equiv M_N\sigma=M_N Y'_w$ (4A)	$\bar{\chi}\equiv M_l\sigma=M_N Y'_w$ (4B)
Transformations	
$\left.\begin{array}{l} \xi=\bar{\chi},\quad \eta=Y/Y_w \\ p\equiv\rho_N u_N^2\sigma^2 P(\xi,\eta) \\ \quad =\rho_N u_N^2 Y_w'^2 P \\ \rho\equiv\rho_N R(\xi,\eta) \\ u=u_N \\ v\equiv u_N\sigma V(\xi,\eta)=u_N Y'_w V \end{array}\right\}$ (5A)	$\left.\begin{array}{l} \xi=\bar{\chi},\quad \eta=Y/Y_w \\ p\equiv\rho_l u_l^2\sigma^2 P(\xi,\eta) \\ \quad =\rho_N u_N^2 Y_w'^2 P/s^4 \\ \rho\equiv\rho_l R(\xi,\eta)=\rho_N R/s^2 \\ u_l=u_N \\ v\equiv u_l\sigma V(\xi,\eta)=u_N Y'_w V/s \end{array}\right\}$ (5B)

Transformed equations of motion[a]

$$\left.\begin{array}{l} (V-\eta)R_\eta+RV_\eta+RV/\eta=-\nu\xi R_\xi \\ \nu RV+(V-\eta)RV_\eta+P_\eta=-\nu\xi V_\xi \\ 2\nu PR+(V-\eta)(RP_\eta-\gamma PR_\eta)=-\nu\xi(PR_\xi-\gamma PR_\xi) \\ \nu\equiv Y_w Y''_w/Y_w'^2 \end{array}\right\} \quad (6)$$

Pressure coefficient[b]

$$c_p\equiv\frac{p_b-p_N}{\rho_N u_N^2/2}=\frac{2}{\gamma M_N^2}\{\gamma M_N^2 Y_w'^2 P_b-1\} \quad (7A)$$

$$c_{pl}\equiv\frac{p_b-p_l}{\rho_N u_N^2/2}=c_p/s^4, \quad (7B)$$

[a] Shock condition and surface condition have the same expressions in the unified, transformed variables. [b] Referred to freestream local pressure p_l.

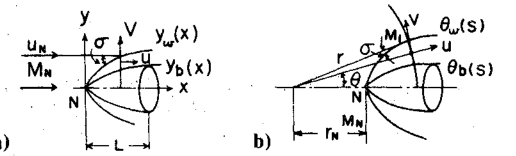

Fig. 1 Hypersonic flow over a slender body: a) parallel flow, b) source flow ($\gamma = 2$).

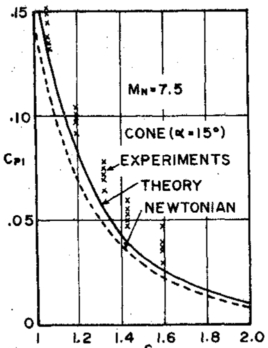

Fig. 2 Pressure distribution along the cone surface in the source flow with $M_N = 7.5$.

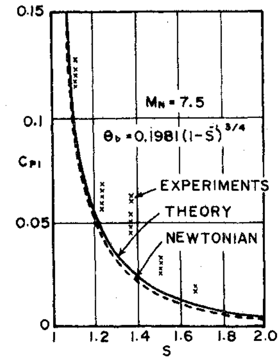

Fig. 3 Pressure distribution along the ¾ power-law nosed cone in the source flow with $M_N = 7.5$.

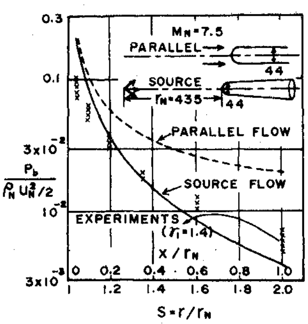

Fig. 4 Pressure distribution along the hemisphere cone in the source flow with $M_N = 7.5$.

corresponding source flow with any γ_l, using the simple relation $c_{pl} = c_p/s^4$, in which the value of c_p must be taken at s where $x/L = 1 - s^{-1}$ holds. In this conversion, it is assumed that the term c_p in the expression of c_{pl} is also insensitive to the difference of γ_l in the source flow problem. Similarly, when the source flow results are known, the corresponding relations for the parallel flow can be obtained by simply inverting the above procedure.

Some practical results of pressure distributions for a cone, a ¾ power-law cone, and a needle-type hemisphere cone in the source flow with $\gamma_l = 1.4$ are obtained from the parallel flow results of Lees and Kubota,[3] and Yasuhara and Watanabe,[1] as follows:

$$\theta_b = 0.2679(1 - s^{-1}); \quad \text{cone}, \ s \equiv r/r_N \tag{8a}$$

$$(c_{pl})_{\text{cone}} = (0.1579 + 0.3304/M_N^2)/s^4 \tag{8b}$$

$$\theta_b = 0.1981(1 - s^{-1})^{3/4}: \ \text{¾ power-law nosed cone} \tag{9a}$$

$$(c_{pl})_{3/4} = \{0.04317(1 - s^{-1})^{-1/2} + 0.5228/M_N^2\}/s^4 \tag{9b}$$

$$\theta_b = d/2r_N = 0.05057, \quad (s > 1) \tag{10a}$$

(needle-type hemisphere cone with the nose diameter of d, and $d/r_N = 0.1011$.):

$$(c_{pl})_{\text{hemi}} = \left\{\frac{0.0950 \times 0.1011}{(1 - s^{-1})} - \frac{0.850}{M_N^2}\right\} \Big/ s^4 \tag{10b}$$

or

$$\frac{p_b}{\rho_N u_N^2/2} = \left\{\frac{0.009605}{(1 - s^{-1})} - \frac{0.850}{M_N^2}\right\} \Big/ s^4 + \frac{2}{\gamma_l M_N^2} s^{-2\gamma_l} \tag{10c}$$

with $\gamma_l = 1.4$.

The pressure distributions for the source flow with $\gamma_l = 1.4$ and $M_N = 7.5$, obtained by applying the present method are shown in Figs. 2-4 as functions of $s = r/r_N$. Figures 2 and 3 show c_{pl}, and Fig. 4 shows the pressure p_b normalized by $\rho_N u_N^2/2$. In these figures, we have also plotted experimental results from Ref. 4, and some of our own measurements.

These experiments were performed using the conical nozzle of the shock tunnel at the Nagoya University, as described in Ref. 4. The freestream Mach number at the nose of bodies was fixed at $M_N = 7.5$. Air was used as the working gas, so that $\gamma_l = 1.4$, and the body geometries are given by functional expressions of θ_b in the preceding section.

Although the theoretical results (which we obtained by using the analogy between source and parallel flow) do not include the effects of viscosity, there is (within the experimental error) good agreement with the measurements for power-law cones. For the hemispherical cones in Fig. 4 there appears to be a systematic crossing of the experimental and theoretical values. This may be explained as follows: the parallel flow result of Lees and Kubota[3] are obtained from the explosion analogy, and in this case the solution is sensitive to the value of γ. This fact, in turn, should influence the extended parallel-source analogy if γ_l is different from 2, because the exact analogy holds only if both γ and γ_l are equal to 2.

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